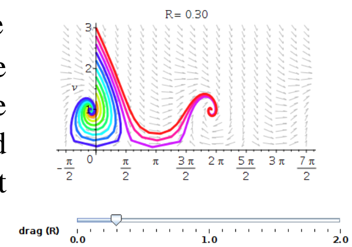
MAT331 Exercises, Fall 2019 *set number* 6

22. (*expires 10/21*) On Oct. 8, we made an animation of solutions to the Phugoid model as the drag parameter R changes. Modify this to produce a plot of the corresponding picture in the (θ, v)-plane where the value of R is controlled by a slider, similar to Exercise 12. Your plot should change as the slider is moved, and look something like the image at right. You might want to refer to the worksheet sliderfit.mw.



23. (*expires 10/21*) In this problem will study the Lotke-Volterra predator-prey equations. In a very simple ecosystem there are two populations, whose numbers at a time t (with t in, say, years) are given by $f(t)$ (foxes) and $r(t)$ (rabbits). The evolution of these quantities obeys the system

$$\begin{cases} \dot{f}(t) &= G_f\, f(t) + E\, f(t)\, r(t), \\ \dot{r}(t) &= G_r\, r(t) - E\, f(t)\, r(t); \end{cases}$$

where G_f and G_r are the growth rates for the foxes and the rabbits, respectively, in the absence of each other. E is the probability of a fatal encounter between a fox and a rabbit (normalized per number of foxes and rabbits).

First, write some words to explain why these equations make sense. Then, fix $G_f = 0.4$, $G_r = 2.4$ (it's notorius that rabbits have the tendency to reproduce quickly) and $E = 0.01$. For a few initial conditions of your choice, plot the trajectories in the (f, r)-plane (say, with $0 \le f \le 1000$ and $0 \le r \le 1000$). For the same initial conditions, plot the actual solutions too (i.e, $f(t)$ against t, and $r(t)$ against t). Write some comments interpreting how the behavior of the solutions relates to what happens to the two species. (Here, to plot $f(t)$ against t, you can use the `scene` argument to `DEplot`, or you can use `dsolve` and maybe `plots[odeplot]`.)

Finally, repeat the same procedure with $G_f = -1.1$. Things change substantially. As above, explain how the solution behavior relates to the populations of foxes and rabbits. What does having $G_f = -1.1$ mean in the context of rabbit and fox populations?

24. (*expires 10/21*) Consider the differential equation corresponding to the the vector field

$$\mathbf{F}(x, y) = \left\langle -x(x^4 + y^4) - y\,,\; x - y(x^4 + y^4) \right\rangle \; .$$

Use Maple to draw the either the direction field or the vector field, together with some well-chosen solution curves. (I would use `DEplot` here, but you can use a combination of `fieldplot`, `dsolve` (with the `numeric` option), `plots[odeplot]`, and `plots[display]` if you prefer.)

Then *prove* that the origin is a global attractor in the future, i.e., for every solution $\mathbf{z}(t) = \big(x(t), y(t)\big)$, we have

$$\lim_{t \to +\infty} \mathbf{z}(t) = \mathbf{0}.$$

Note: The proof is not long, but requires a mathematical argument, not a maple calculation. The proof may depend on something you calculated in maple, but more will be needed. Polar coordinates can be your friend.